QUANTUM ATOMIC CLOCK SYNCHRONIZATION: AN ENTANGLED CONCEPT OF NONLOCAL SIMULTANEITY

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We demonstrate that two spatially separated parties (Alice and Bob) can utilize shared prior quantum entanglement, as well as a classical information channel, to establish a synchronized pair of atomic clocks. Within their common inertial frame, Alice and Bob execute a sequence of local measurements and classical communiqués, which extract the clock pair and guarantees their synchrony. In contrast to classical synchronization schemes, our protocol is independent of Alice or Bob's knowledge of their relative locations or of the properties of the intervening medium.

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In the *Special Theory of Relativity,* there are two common methods for synchronizing a pair of spatially separated clocks, A and B, which are at rest in a common inertial frame. The usual procedure is that of *Einstein Synchronization,* which involves an operational line-of-sight exchange of light pulses between two observers, say Alice and Bob, who are co-located with the clocks A and B, respectively [1]. A less commonly used *Clock Synchronization Protocol (CSP)* is that of Eddington, namely, *Slow Clock Transport (SCT).* In the SCT scheme, the two clocks A and B are first synchronized locally and then they are transported adiabatically (infinitesimally slowly) to their final separated locations in the common inertial frame [2]. These two rather different protocols lead to the same physical consequences, under certain conventionality assumptions of the isotropy of the one-way speed of light [3]. In our current work we would like to propose a third CSP, which utilizes quantum informatics techniques that exploit the resource of shared prior entanglement between the two synchronizing parties.

Our proposed method of *Quantum Atomic Clock Synchronization (QuACS)* has many elements in common with Ekert's, entanglement-enabled, quantum key-distribution [4]. In the Ekert scheme, Alice and Bob initially share only prior-entangled qubit pairs, and the key does not yet exist. The key is then extracted from the ensemble of entangled pairs through a series of measurements and classical communiqués. In other words, no key is ever physically transported between Alice and Bob, which is the basis for the security of this distribution system. Likewise, for our QuACS protocol, there initially exist no clocks, but rather only shared entanglement. The synchronized clocks are extracted from the prior entanglement via the measurements performed by Alice and Bob, along with their subsequent classical communications. In this way our QuACS scheme establishes synchrony without having to transport timing information between Alice and Bob, in analogy to that feature of Ekert key distribution. In contradistinction, for the classical Einstein and SCT synchronization schemes, synchrony information must transmitted by Alice to Bob over some classical channel, which limits the accuracy of the synchronization. Let us now see how our QuACS protocol works.

Let us first review how an atomic clock operates, in the language of quantum information theory. An atomic clock consists of an ensemble of identical two-level systems (qubits) whose temporal evolution rate is taken as the time standard. For example, the second is defined as exactly 9,192,631,770 periods of oscillation corresponding to the hyperfine-transition (radio) frequency in the ground state of the Cs^{133} atom [5]. The fact that this frequency is identical for all Cs^{133} atoms, which are sufficiently isolated from the environment, allows anyone to establish a Cs^{133} time standard of comparable accuracy. In general, any set of identical qubits may be used as the time standard in a temporal interferometer, which employs the Ramsey method of separated oscillatory fields [6].

Specifically, let us suppose the qubit time standard has energy eigenstates $|0\rangle$ and $|1\rangle$, which evolve as usual under the unitary transform U(t), namely [7],

$$U(t) = \begin{pmatrix} e^{-i\hbar E_1 t} & 1 \\ 1 & e^{-i\hbar E_0 t} \end{pmatrix} \quad \Rightarrow \quad |0\rangle \rightarrow e^{-i\hbar E_0 t}|0\rangle \quad \text{and} \quad |1\rangle \rightarrow e^{-i\hbar E_1 t}|1\rangle, \tag{1}$$

where E_0 and E_1 are the eigen-energies of $|0\rangle$ and $|1\rangle$ respectively, and $\text{ⱨ} \equiv 1/\hbar$ with $\hbar$ being Dirac's constant. To construct an atomic qubit clock via the Ramsey method [6], we apply a $\pi/2$ radio-wave pulse (Hadamard transformation) to an ensemble of N identical qubits in state $|0\rangle$ at some time $t = 0$. This unitary one-qubit Hadamard transform generates an ensemble of equal-superposition states, $|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}$, which then evolve unitarily as,

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to \frac{1}{\sqrt{2}}(|0\rangle + e^{i\Omega t}|1\rangle) \equiv \frac{1}{\sqrt{2}}(|0\rangle e^{-i\Omega t/2} + e^{i\Omega t/2}|1\rangle), \tag{2}$$

up to an unobservable overall phase factor. Here, $\Omega \equiv \text{ⱨ}(E_0 - E_1)$ is the frequency standard, which is known exactly since it *defines* the unit of time.

After a time t, we now apply a second Hadamard transform ($\pi/2$ pulse) to the evolving ensemble of qubits. This transform again maps $|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}$, as well as now $|1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$, to yield a final clock wave function,

$$|\psi(t)\rangle = \cos(\tfrac{1}{2}\Omega t)\;|0\rangle - i\sin(\tfrac{1}{2}\Omega t)\;|1\rangle\;. \tag{3}$$

At this point, we perform a final projective counting measurement of the qubit populations in either $|0\rangle$ or $|1\rangle$, which occur with probabilities P_0 or P_1, given by,

$$P_0 = |\langle 0|\psi(t)\rangle|^2 = \tfrac{1}{2}(1 + \cos\Omega t) \quad \text{and} \quad P_1 = |\langle 1|\psi(t)\rangle|^2 = \tfrac{1}{2}(1 - \cos\Omega t)\;, \tag{4}$$

where $P_0 + P_1 = 1$, from unitarity. Now by monitoring the oscillations of either P_1 or P_2 ,as a function of time, we get an estimate of the clock phase $\Omega t \bmod 2\pi$ that is an equally good estimate of t, since the standard Ω is known with infinite precision.

Now that we have completed our review of the operation of an atomic clock, let us consider our proposed set up for quantum clock synchronization. First we would like to consider a particular form of prior entanglement between pairs of qubits shared by Alice and Bob. Recall that our atomic clock qubits, like any two-level system, can be modeled by the SU(2) angular momentum algebra for spin-1/2 particles. Focussing on a specific shared qubit pair in the ensemble held by Alice and Bob, let us call their vector spin-1/2 operators $\hat{\mathbf{S}}_A$ and $\hat{\mathbf{S}}_B$, each operating on a 2D Hilbert space $\mathcal{H}$. The total spin is $\hat{\mathbf{S}} = \hat{\mathbf{S}}_A + \hat{\mathbf{S}}_B$, which operates on the 4D direct-product space $\mathcal{H} \otimes \mathcal{H}$. A basis for this product space is given by the eigenstates of the z components of $\hat{\mathbf{S}}_A$ and $\hat{\mathbf{S}}_B$, namely $\{|0\rangle_A|0\rangle_B$, $|0\rangle_A|1\rangle_B$, $|1\rangle_A|0\rangle_B$, $|1\rangle_A|1\rangle_B\}$. Following the rules for addition of angular momentum [7], we may construct the four simultaneous eigenstates of $\hat{S}_z$ and $\hat{\mathbf{S}}^2$ as,

$$|\psi_{0,0}\rangle = \tfrac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right), \quad |\psi_{1,-1}\rangle = |0\rangle_A|0\rangle_B, \tag{5a}$$

$$|\psi_{1,0}\rangle = \tfrac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right), \quad |\psi_{1,1}\rangle = |1\rangle_A|1\rangle_B, \tag{5b}$$

where the subscripts {j,m} indicate the total and z-component angular momentum eigenvalues, respectively. The states with j = 0 and j = 1 are called the singlet and triplet states, respectively. Important for our argument here is use of the entangled singlet state $|\psi_{0,0}\rangle$, which spans a spherically symmetric 1D subspace of SU(2)⊗SU(2), corresponding to zero total angular momentum. This singlet state is a "dark state" that does not evolve in time, since its unitary temporal evolution follows,

$$U(t)\otimes U(t)|\psi_{0,0}\rangle = \tfrac{1}{\sqrt{2}}\left(e^{-i\hbar E_0 t}|0\rangle_A e^{-i\hbar E_1 t}|1\rangle_B - e^{-i\hbar E_1 t}|1\rangle_A e^{-i\hbar E_0 t}|0\rangle_B\right) = e^{-i\hbar(E_0+E_1)t}|\psi_{0,0}\rangle, \quad (6)$$

which is equal to $|\psi_{0,0}\rangle$ up to an unobservable overall phase. (In general, for *any* one-qubit unitary transformation U, we have $U\otimes U\,|\psi_{0,0}\rangle = (\det U)\,|\psi_{0,0}\rangle$.) Hence the pair of shared, "pre-clock" ensembles of entangled Bell states of this form can be said to be "idling" as far as their overall temporal evolution is concerned.

We imagine then that Alice and Bob share a ensemble of a large number N of such entangled pairs, labeled in order as n = 1, 2, 3, ... N. (The qubits could be arranged, say, in an ion or atom trap where each qubit's label is identified by its location in an array.) Now to start the clocks at some time $t = t_A^0 = 0$, Alice *simultaneously* measures all of her pairs in the orthogonal basis $\{(|0\rangle+|1\rangle)/\sqrt{2}, (|0\rangle-|1\rangle)/\sqrt{2}\}$. Since in this basis the singlet state can be written,

$$|\psi_{0,0}\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_A \tfrac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_B - \tfrac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_A \tfrac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_B\,, \quad (7)$$

each pair in the ensemble randomly collapses *simultaneously* at A and B into either

$$\left|\psi^{I}\right\rangle = \tfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)_A \tfrac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)_B \quad \text{or} \quad \left|\psi^{II}\right\rangle = \tfrac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)_A \tfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)_B \ , \tag{8}$$

with probability 1/2. Alice's measurement has removed the singlet-state invariance under temporal evolution, and thus the A and B clocks begin to evolve in time as per Eq. (1)—both starting synchronously at time $t=0$ in Alice and Bob's shared inertial frame. The idling clocks are now engaged, and they begin to run.

Comparing the entangled-clock wavefunctions in Eq. (8) with the ordinary clock formula in Eq. (2), we see that Alice's measurement effectively reproduces the result of the first one-clock Hadamard transform in the Ramsey scheme. However, the resultant is now four distinct clocks: two sub-ensembles of half of Alice's collection of qubits each form separate clocks that are π out of phase with each other, with a similar situation for Bob. In addition, each of Bob's sub-ensembles is running in synchrony with one of Alice's, but they do not yet know which is which.

The next step in our QuACS protocol is for Alice and Bob to perform a Hadamard ($\pi/2$ pulse) transform on their evolving qubit ensembles at times t_A and t_B, respectively. The evolving wavefunctions in Eq. (8) then transform as,

$$\left|\psi^{I}_{\{A,B\}}\right\rangle \equiv \tfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)_{\{A,B\}} \ \to \ \cos\!\big(\tfrac{1}{2}\Omega t_{\{A,B\}}\big)\ |0\rangle \ - \ i\sin\!\big(\tfrac{1}{2}\Omega t_{\{A,B\}}\big)\ |1\rangle \ , \tag{9a}$$

$$\left|\psi^{II}_{\{A,B\}}\right\rangle \equiv \tfrac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)_{\{A,B\}} \ \to \ -i\sin\!\big(\tfrac{1}{2}\Omega t_{\{A,B\}}\big)\ |0\rangle \ + \ \cos\!\big(\tfrac{1}{2}\Omega t_{\{A,B\}}\big)\ |1\rangle \ . \tag{9b}$$

If Bob now carries out the usual projective counting measurement onto either the $|0\rangle$ or $|1\rangle$ states for his *entire* ensemble of qubits, he gets no timing information. For example, if he projects onto the $|0\rangle$ ground state at some time $t = t_B^0 > t_A^0 \equiv 0$, then he will get either $P_B^I = \left|\langle 0 | \psi_B^I \rangle\right|^2 = \frac{1}{2}\left[1 + \cos\Omega\left(t_B + t_B^0\right)\right]$ or $P_B^{II} = \left|\langle 0 | \psi_B^{II} \rangle\right|^2 = \frac{1}{2}\left[1 - \cos\Omega\left(t_B + t_B^0\right)\right]$, randomly, with probability 1/2. (The same holds for projection onto $|1\rangle$). Thus, his ensemble average will be a flat, constant 1/2 in time without the required clocking oscillations. For Bob to extract a clock, a classical message from Alice is now required.

So let us suppose that Alice now post selects from her entire ensemble the sub-ensemble of, say, Type I qubits. Since the qubits are ordered, she can then tell Bob which subset of his ensemble are also of Type I by broadcasting their ordinal labels via any form of classical communiqué. Bob is then able to extract, beginning at some time $t = t_B^0 > t_A^0 \equiv 0$, his own Type-I sub-ensemble, which is guaranteed to be running exactly π out of phase with Alice's. The couple's two type I clocks are thus anti-correlated, just as in Ekert's quantum key distribution scheme [4]. Now, by monitoring the temporal oscillations of his type I clock, Bob is assured by our protocol that his clock phase Ωt_B is related to Alice's phase Ωt_A, via $\Omega t_B = (\Omega t_A - \pi) \bmod 2\pi$. From this information, Bob may prepare a third π-phase-shifted clock, using local classical synchronization, that is guaranteed to be in synchrony with Alice's Type-I clock, up to modulo 2π. In other words, Alice and Bob now have clocks that are "ticking" in unison.

For some applications, such as satellite-based *Very Long Base-Line Interferometry (VBLI)* [8], the fact that Alice and Bob's are phase locked only up to modulo 2π is sufficient. However, there are other applications, such as the synchronization of atomic clocks in the *Global Positioning System (GPS)* constellation [9], where it is important to have an absolute reference point or origin of time. For these applications, it is a simple matter to adapt our QuACS protocol to one that reveals a shared time origin. Let us suppose that, in addition to the standard clock qubits that run at the defined frequency Ω, we have an additional set of identical qubits all with a slightly shifted frequency Ω', such that $\Omega' - \Omega \equiv \Delta\Omega$. For example, if Ω corresponds to the two-level hyperfine (qubit) transition of the standard Cs^{133} clock atom, then Ω' could correspond to the same transition in the long-lived radioactive isotope Cs^{135}, which is slightly different from Ω due to the isotope shift [10].

So now Alice and Bob prepare two sets of two ensembles of entangled pairs in the singlet state: one with frequency Ω and the other with Ω'. As before, Alice performs a simultaneous measurement on all the atoms in both of her ensembles using the orthogonal basis $\{(|0\rangle+|1\rangle)/\sqrt{2}, (|0\rangle-|1\rangle)/\sqrt{2}\}$. This starts the evolution of both types of clocks. The protocol is exactly the same as before, and therefore Alice transmits information about the Type I and Type I′ sub-ensemble clocks to Bob via a classical channel. Bob is then able to extract two Type-I and Type-I′ clocks, running at slightly different rates, which are in anti-synchrony with Alice's corresponding clocks. By monitoring the qubits in, say, the $|0\rangle$ state of his two clocks, Bob observes the following two evolution probabilities, $P_B = \frac{1}{2}\left[1+\cos\Omega\left(t_B + t_B^0\right)\right]$ and $P_B' = \frac{1}{2}\left[1+\cos\Omega'\left(t_B + t_B^0\right)\right]$. Bob now may subtract these two signals to get a difference function,

$$f(t_B) = \sin\left[\tfrac{1}{2}\Delta\Omega\left(t_B + t_B^0\right)\right]\sin\left[\tfrac{1}{2}(\Omega + \Omega')\left(t_B + t_B^0\right)\right] , \tag{10}$$

whose first term is a slowly varying beat envelope that oscillates at frequency $\Delta\Omega$, and whose second term oscillates rapidly at the average of the two clock frequencies, $(\Omega + \Omega')/2$. From the beat envelope function, $e(t_B) = \sin\left[\frac{1}{2}\Delta\Omega\left(t_B + t_B^0\right)\right]$, Bob can determine an origin of time in coincidence with Alice's, if they both arrange in advance that the complete set of classical communiqués takes place in a time short enough such that $\frac{1}{2}\Delta\Omega t_B^0 < \pi$. Bob can then be sure that the slowly varying beat envelope is still in its first half-cycle of oscillation, which allows him to evaluate the measured function $e(t_B)$ at $t_B = 0$ and invert it within a single branch of the arcsine function to get $t_B^0 = 2\arcsin\left[e(0_B)\right]/\Delta\Omega$, uniquely. Bob now knows that the origin of his time axis, which is entirely conventional, must be translated backwards in time by an amount t_B^0, so that his origin of time corresponds with Alice's. Alice and Bob now have exact synchrony in their spatially separated clocks, which are at rest in their common inertial frame.

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REFERENCES

1. A. Einstein, Ann. D. Physik **17**, 891 (1905); English translation in, *The Collected Papers of Albert Einstein. The Swiss Years: Writings, 1900–1909,* Vol. 2 (Princeton University Press, Princeton, NJ, 1989), pp. 140–171.

2. A. S. Eddington, *The Mathematical Theory of Relativity,* 2nd ed., (Cambridge University Press, Cambridge, 1924).

3. R. Anderson, I. Vetharaniam, and G. E. Stedman, Phys. Rep. **295**, 93 (1998).

4. A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).

5. C. Audoin, Metrologia **29**, 113 (1992).

6. N. F. Ramsey, Molecular Beams (Oxford University Press, Oxford, UK, 1969), Sec. V.4.

7. E. Merzbacher, *Quantum Mechanics,* 2nd ed. (John Wiley & Sons, NY, 1970), Sec. 7.6.

8. G. S. Levy, *et al.,* Acta Astronaut. **15**, 481 (1987).

9. National Research Council Staff, *The Global Positioning System: A Shared National Asset* (National Academy Press, Washington, DC, 1995).

10. I. I. Sobelman, *Atomic Spectra and Radiative Transitions,* 2nd ed. (Springer-Verlag, Berlin, 1992), Sec. 6.2.5.